Impedance Tapering Effects on "Low-Q" SAB Band-Pass Active-*RC* Filter

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***Abstract*—The design procedure of low-sensitivity active resistance-capacitance (RC) allpole filters, using impedance tapering, has already been published. The low-sensitivity filter sections already described in publication are class 4 (TT-SABB) Sallen and Key sections. In this paper class 3 (SAB) sections for low pole-Q realization are considered. Impedance tapering is applied on L-sections, which are situated in negative-feedback of operational amplifier in open-loop mode. L-sections are impedance scaled upwards, from the driving source to the negative amplifier input. Second-order band-pass filter is considered. Pole-Q factors for low-Q building blocks take their values up to, say, 5. The sensitivity to component tolerances of the circuit is shown to be small for any type of impedance tapering regardless of the gain-sensitivity product (GSP) value.**

***Index Terms*—Allpole filters, biquadratic active filters, class 3 active filters for low-Q realization, low-sensitivity active filters.**

1. INTRODUCTION

A procedure for design of low-sensitive allpole filters has already been presented in [1] and [2]. The filter circuits in [1] and [2] are class 4 Sallen and Key [3] and the design presented is based on "impedance tapering".

In this paper we apply impedance tapering to class 3 allpole active-*RC* filters for low pole-Q factor realization. To choose "low-Q" circuit, there is only one criteria, i.e. the value of Q-pole factor, q_p (see [5]) which has to be smaller than say 5.

Based on the class 3 filter circuits shown in [4] and [5], having an *RC* ladder network in the negative feedback of an operational amplifier, impedance tapering is applied. The adequate L-sections of an *RC* ladder in a feedback loop are thus successively ipedance scaled upwards, from the driving source to the negative amplifier input. Second-order band-pass filter is considered. Impedance tapering is applied on "low-Q" SAB filter blocks. On one example Monte Carlo analysis was performed with simulation program PSPICE to examine the sensitivity of the filter's transfer function to component tolerances. Simple Voltage-Controlled-Voltage-Source (VCVS) has been used as a substitution for ideal operational amplifier. Sensitivity to active elements in the filter circuit is represented by the gain-sensitivity product (GSP). It is shown that the sensitivity of the filter's characteristics on component tolerances is small for any type of impedance tapering regardless of the GSP value.

2. DEFINITION OF SENSITIVITY

Relative sensitivity of a function $F(x)$ to variations of variable x is defined as

$$S_x^{F(x)} = \frac{dF/F}{dx/x} = \frac{dF(x)}{dx}\frac{x}{F(x)} = \frac{d[\ln F(x)]}{d[\ln x]}. \tag{2.1}$$

Consider the transfer function $T(s)$ of the second-order, allpole band-pass filter given with eq. (3.1). Filter coefficients a_i of the polynomial $D(s)$ are available from any filter handbook.

Relative change of $T(s)$ to the variation of its coefficients a_i is

$$\frac{\Delta T(s)}{T(s)} = \sum_{i=0}^{n} S_{a_i}^{T(s)} \frac{\Delta a_i}{a_i} \tag{2.2}$$

where $S_{a_i}^{T(s)}$ is *sensitivity to coefficient variations* and is dependent only on a value of coefficients a_i and frequency ω. Coefficient variation is represented with

$$\frac{\Delta a_i}{a_i} = \sum_{\mu=1}^{r} S_{R_\mu}^{a_i} \frac{\Delta R_\mu}{R_\mu} + \sum_{\nu=1}^{c} S_{C_\nu}^{a_i} \frac{\Delta C_\nu}{C_\nu} \tag{2.3}$$

On the other hand *coefficient-to-component sensitivities* $S_x^{a_i}$, where x represent each of the component types, are dependent on the realisation of the filter circuits and can be reduced with non-standard filter design as shown in [1]. The sensitivity reduction is achieved applying "impedance tapering" instead of standard filter design techniques.

3. SECOND-ORDER LOW-Q BAND-PASS FILTER CLASS 3

As representative example we consider the second-order band-pass (R) filter shown in Fig. 1 as in [5]. It has a ladder-circuit in negative feedback of an operational amplifier in open-loop mode and is known as class 3 "low-Q" band-pass filter. Note that the operational amplifier has its positive pin grounded.

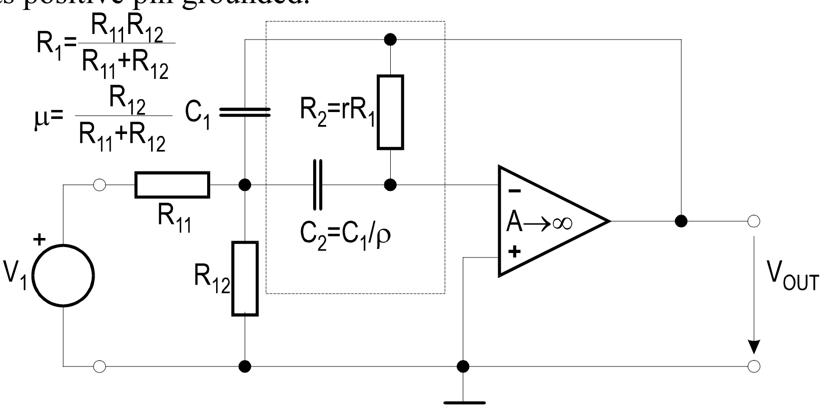

Fig. 1 Second-order Band-pass Filter Class 3 (Low-Q Realisation)

The voltage transfer $T(s)$ function for this circuit expressed in terms of the coefficients a_i is given by:

$$T(s) = \frac{V_{OUT}}{V_1} = \frac{N(s)}{D(s)} = \frac{K a_1 s}{s^2 + a_1 s + a_0}, \tag{3.1}$$

and in terms of the pole frequency ω_p and pole Q, q_p by

$$T(s) = \frac{K' \omega_p s}{s^2 + \frac{\omega_p}{q_p} s + \omega_p^2}, \tag{3.2}$$

where $K = q_p K' = q_p \cdot \mu \cdot \sqrt{\frac{R_2 C_2}{R_1 C_1}}$ and

$$a_0 = \omega_p^2 = \frac{1}{R_1 R_2 C_1 C_2},$$

$$a_1 = \frac{\omega_p}{q_p} = \frac{C_1 + C_2}{R_2 C_1 C_2},$$

$$q_p = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2)}. \tag{3.3}$$

To achieve gain K in pass band, where K is gain factor specified by the filter designer, a value of μ must be given:

$$\mu = \frac{K}{q_p \sqrt{\frac{R_2 C_2}{R_1 C_1}}} . \tag{3.4}$$

If the desired value μ is less than unity, then the specified gain K can be tuned with a resistive voltage divider at the input of the network, consisting of R_{11} and R_{12}, as shown in Fig. 1. For a value of $\mu>1$ it is possible to make output voltage-level transformation, which is not presented in this paper (see [1] and [4]).

The sensitivity of a_0 to all RC components is –1, thus $\Delta a_0/a_0$ can be decreased only technologically. For the sensitivity of a_1 to tolerances of passive components, we readily obtain expressions given in the first column of Table 1. Note that the coefficient a_1 sensitivities to R_1 and R_2 are 0 and –1, respectively. The only improvement can be done with the sensitivites to capacitors. They are all proportional to the pole Q, q_p. Thus, one does well when select the filter type with the lowest pole Q's, for a given application.

Reffering to Fig. 1, the second L-section in the feedback loop comprising R_2 and C_2 (inside the rectangle) can be impedance scaled upwards in order to minimize the loading on the first, i.e. R_1 and C_1. Letting

$$R_1=R; \quad C_1=C; \quad R_2=rR; \quad C_2=C/\rho \tag{3.5}$$

we obtain the sensitivity relations given in second column of Table 1. Observing those sensitivities, one can see that some of the sensitivities are proportional to ρ and some to ρ^{-1}, and setting $\rho=1$ provides an optimum compromise.

Table 1 Sensitivity of a_1 to Component Variations of "Low-Q" Second-Order Band-pass Filter Class 3

x	$S_x^{a_1}$	
		$R_1=R$; $C_1=C$ $R_2=rR$; $C_2=C/\rho$
R_1	0	0
R_2	-1	-1
C_1	$-q_p\sqrt{\frac{R_1C_2}{R_2C_1}}$	$-q_p\frac{1}{\sqrt{r\rho}}$
C_2	$-q_p\sqrt{\frac{R_1C_1}{R_2C_2}}$	$-q_p\sqrt{\frac{\rho}{r}}$

Design equations for tapered second-order band-pass filter follow. Its transfer function is given by (3.1) to (3.3). With the tapering factors in (3.5) and with

$$\omega_0 = \frac{1}{RC} \tag{3.6}$$

we obtain for the coefficients of $T(s)$

$$a_0 = \omega_p^2 = \frac{\rho}{r}\cdot\omega_0^2; \quad a_1 = \frac{\omega_p}{q_p} = \omega_0 \frac{\rho+1}{r} . \tag{3.7}$$

From K, a_0 and a_1, which are given by filter specifications, we must determine ω_0, ρ, r and μ. Therefore we use the following expressions:

$$r = \frac{\omega_0^2}{a_1\omega_0 - a_0}, \quad \rho = \frac{ra_0}{\omega_0^2} = \frac{a_0}{a_1\omega_0 - a_0} . \tag{3.8}$$

Since r and ρ must both be positive, the denominator of (3.8) must be larger than zero, resulting in the following constraint

$$\omega_0 > \frac{a_0}{a_1} = \omega_p q_p . \qquad (3.9)$$

Example: Consider the following practical example. Suppose that

$$\omega_p = 2\pi \cdot 86\,\text{kHz}; \quad q_p = 5; \quad C = 500\,\text{pF}. \qquad (3.10)$$

During the design process, various ways of impedance tapering has been applied and the resulting component values are presented in Table 2. On the resulting filters Monte Carlo runs with 5% gauss-distribution, zero-mean resistors and capacitors were carried out and presented in Fig. 3.

Table 2 Component Values of Impedance Tapered "low-Q" circuits with ρ=0.1, 1, 3, 10, 25 and q_p=1, 3, 5.
(Resistors in [kΩ], Capacitors in [pF])

Nr.	q_p	Impedance Tapered Filter	R_1	R_2	r	C_1	C_2	ρ	GSP
1.	1	ρ=0.1	0.336	4.071	12.1	500	5000	0.1	11
2.		ρ= 1	1.85	7.402	4	500	500	1	2
3.		ρ= 3	2.775	14.81	5.33	500	166.7	3	1.33
4.		ρ=10	3.365	40.71	12.1	500	50	10	1.1
5.		ρ=25	3.558	96.23	27.4	500	20	25	1.04
6.	3	ρ=0.1	0.113	12.21	108.9	500	5000	0.1	99
7.		ρ= 1	0.617	22.21	36	500	500	1	18
8.		ρ= 3	0.925	44.42	48	500	166.7	3	12
9.		ρ=10	1.122	122.1	108.9	500	50	10	9.9
10.		ρ=25	1.186	288.7	243.4	500	20	25	9.36
11.	5	ρ=0.1	0.067	20.36	302.5	500	5000	0.1	275
12.		ρ= 1	0.370	37.01	100	500	500	1	50
13.		ρ= 3	0.555	74.03	133.3	500	166.7	3	33.33
14.		ρ=10	0.673	203.6	302.5	500	50	10	27.5
15.		ρ=25	0.712	481.2	676	500	20	25	26

For a given value of capacitive scaling factor ρ and pole Q's value q_p, resistive scaling factor r can be calculated using

$$r = q_p^2 \left(1 + \rho + \frac{1}{\rho}\right) \qquad (3.11)$$

and the GSP using

$$GSP = q_p^2 \left(1 + \frac{1}{\rho}\right). \qquad (3.12)$$

Plots of r and GSP versus ρ with varying pole-Q factors are shown in Fig. 2. Note that GSP is proportional to the squared value of q_p and has no minimal value. Value of r is also proportional to the squared value of q_p, but has minimum when ρ=1, i.e.

$$r_{\min} = r\big|_{(\rho=1)} = 4q_p^2 . \qquad (3.13)$$

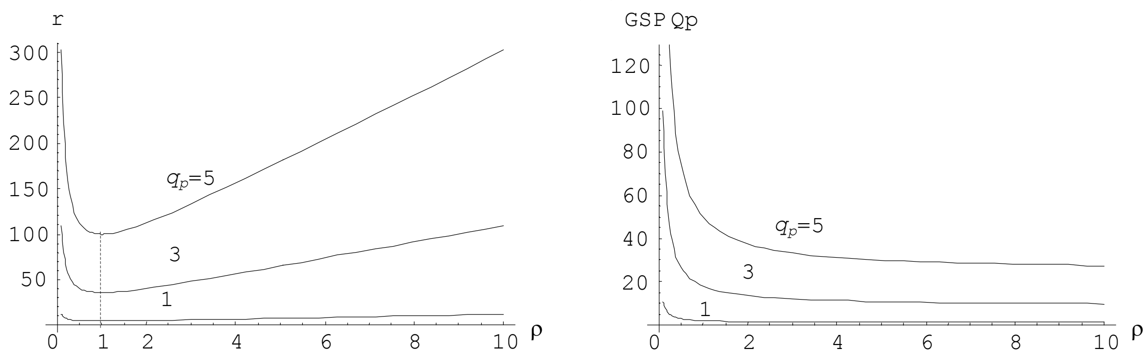

Fig. 2 Plot of r and GSP versus ρ with pole Q's varying from 1 to 5.

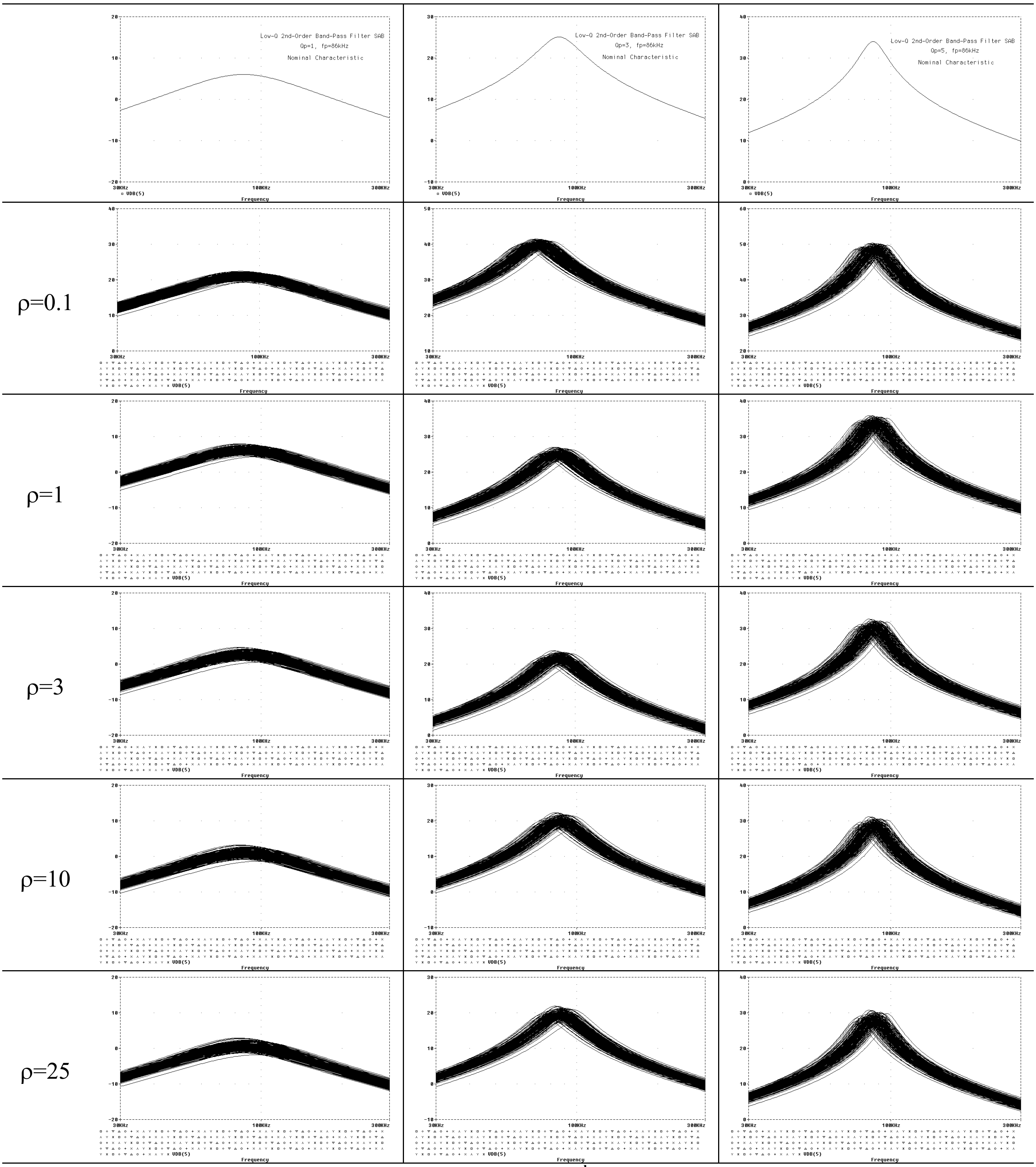

Fig. 3 Monte Carlo response plots of impedance-tapered 2nd-Order "Low-Q" BP filters given in Table 2

Observing Monte Carlo runs in Fig. 3 it can be concluded that by tapering only resistors with equal capacitor values (ρ=1), the filter's characteristic shows somewhat lower sensitivity to component tolerances of the circuit, when compared with ρ>1-tapered circuits. As stated before, this can also be concluded observing the sensitivities in Table 1. Although circuits with ρ>1 and $r>r_{min}$ have lower GSP, they do not have significant decrease in components' variations sensitivity. From the technological realizability point, a big value of r is not preferable, thus taking ρ=1 seem to be a good choice. The problem arises for greater values of q_p, for example q_p=5 yields minimium value of r_{min}=100.

Thus, in summary, for the general second-order allpole band-pass low-Q filter Class 3, resistive impedance tapering with equal capacitors (ρ=1), provide circuits with minimum sensitivity to the component tolerances and technologically realisable resistor values.

3.1 Impedance Tapering of Resistors with ρ=1

As discussed above, the design referring to the circuit in Fig. 1, can be done by the following step-by-step design procedure, for given filter specifications, i.e. K, ω_p and q_p:

i) For given ρ from (3.11) calculate r: Let ρ=1, thus from (3.13) $\rho = 4 \cdot q_p^2 = 4 \cdot 5^2 = 100$.

ii) Calculate ω_0: $\omega_0 = \sqrt{r} \cdot \omega_p = \sqrt{100} \cdot 2\pi \cdot 86\text{kHz} = 5.4 \cdot 10^6 \text{rad/s}$.

iii) Select C_1 and compute R_{11}, R_{12} and R_2:

Let C=500pF thus C_1=C_2=C=500pF, $R = (\omega_0 C)^{-1} = (5.4 \cdot 10^6 \cdot 500 \cdot 10^{-12})^{-1} = 370.13\Omega$. For given pass-band gain K=5 the value $\mu = K/(q_p \beta^- \sqrt{r}) = 5/(5 \cdot 1 \cdot \sqrt{100}) = 0.1 < 1$. Instead of R_1=R=370.13Ω there is voltage attenuator at signal input consisting of R_{11}=R_1/μ= 370.13Ω/0.1= 3.7kΩ and R_{12}=R_1/(1-μ)=370.13Ω/0.9=411.25Ω. Then R_2=rR=37.01kΩ.

iv) Compute GSP: $GSP = q_p^2 (1+1) = 2 \cdot 5^2 = 50$.

The resulting circuit values are given in the row 12) of Table 2.

4. CONCLUSIONS

A procedure for the design of low-sensitive active resistance-capacitance (*RC*) allpole filters of second- and third-order has already been published [1]. In this paper a procedure for design of band-pass 2nd-order "low-Q" SAB circuit as one in [5] is presented. In our example, because there are not very much degrees of freedom available, "impedance tapering" of "low-Q" SAB circuit is in calculating resistive scaling factor r for given value of capacitive scaling factor ρ. It was shown that for ρ=1 value of r is minimal, but still very large for greater values of pole-Q factor, q_p. Also value of GSP does not play important role in the filter's sensitivity. Resistive impedance tapering with equal capacitors (ρ=1) provide technologically realisable circuits with minimum sensitivity to the component tolerances.

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