

# Low-Sensitivity Active-RC Filters Using Impedance Tapering of Symmetrical Bridged-T and Twin-T Networks

Dražen Jurišić and Neven Mijat

Department for Signal and Information Processing  
University of Zagreb  
Unska 3, 10000 Zagreb, Croatia  
[drazen.juriscic | neven.mijat]@fer.hr

George S. Moschytz

Institute for Signal and Information Processing, ISI  
Swiss Federal Institute of Technology, ETH  
Sternwartstrasse 7, 8092 Zürich, Switzerland  
moschytz@isi.ee.ethz.ch

**Abstract**— In this paper we introduce a new design procedure for low-sensitivity filter sections, which have a symmetrical passive-RC network in the operational amplifier (opamp) feedback loop. In the design procedure we apply impedance scaling to the symmetrical bridged-T and twin-T networks; they become "potentially symmetrical" [1]. The design of low-sensitivity allpole active-RC filters, which have an RC-ladder network in the opamp feedback loop, has already been published [2]. There, successive L-sections of the ladder structure are impedance scaled upwards, from the driving source to the positive opamp input; we refer to it as "impedance tapering". In both cases we reduce the filter's magnitude sensitivity to variations of the passive components of a circuit. The new design concept will be demonstrated by designing two very popular and often used filter sections: a band-pass realized by the Deliyannis SAB, and a band-rejection filter with a Twin-T. The sensitivity analysis is examined analytically and double-checked using PSpice Monte Carlo runs.

## I. INTRODUCTION

In this paper we present a low-sensitivity design procedure for two very often-used 2<sup>nd</sup>-order active-RC single-opamp filter sections. One is used for the realization of a band-pass (BP) transfer function and is known as a Deliyannis section. It is realized by a bridged-T RC-network in the negative feedback loop and belongs to "class-3" networks [3]. The other is a band-rejection (BR) section with a Twin-T RC-network in the positive feedback loop and belongs to the "class-4". Both are described in [1][4] and are based on physically symmetrical passive RC networks. The newly introduced design concept reduces the sensitivity to the passive components of the circuit of those two filter sections, making them even more attractive for the realization of filter circuits. In the new design procedure, the topology and component count remain the same, we just judiciously select the component values, in order to decrease the component tolerance sensitivity.

The design method is based on impedance scaling of one half of a symmetrical passive RC-network, in order to maximize its pole Q factor. It is well known that the pole Q,  $\hat{q}$  of a passive RC network is upper limited with the value of (never accessible) 0.5; if we want to reach it we will need to realize an infinite ratio of two components [1]. Note, that we use the symbol "hat" on the top of any passive-network parameter. Design of the passive RC network such that its pole Q is as close to 0.5 as possible, is very valuable due to several reasons presented in [4] (pp. 315-339).

## II. SELECTIVITY OF PASSIVE RC NETWORKS

One very useful network characteristic in connection with RC networks is that of symmetry. It is well known that physically and electrically symmetrical networks (and reciprocal—this property is

generally fulfilled for passive networks ['generally' because it does not hold for the 'passive' gyrator, for example]), can be split into two identical halves. Then, we can apply Bartlett's bisection theorem to readily calculate open-circuit impedances [1].

The process of deriving a so-called "potentially symmetrical" bridged-T network is shown in Fig. 1 if we scale the impedance of one half of the network (e.g. right) by a constant factor  $\rho$ . The transfer function of the bridged-T in Fig. 1 is given by [1][4]:

$$\hat{t}_{32}(s) = \frac{V_2}{V_3} = k_{32} \frac{s^2 + (\omega_z / q_z) \cdot s + \omega_z^2}{s^2 + (\omega_p / \hat{q}) \cdot s + \omega_0^2}, \quad (1)$$

where  $k_{32}=1$ ,  $\omega_0=\omega_z$ ,  $\hat{q} < q_z$  and

$$\omega_0 = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}; \quad q_z = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2)}; \quad \hat{q} = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2) + R_2 C_2}. \quad (2)$$

The transfer function in (1) has the characteristic of the "Frequency Rejection Network" (FRN) [1][3]. For the "symmetrical" bridged-T in Fig. 1(a) we have:

$$\omega_z = \omega_0 = (RC)^{-1}; \quad q_z = 1; \quad \hat{q} = 1/3. \quad (3)$$

Deriving the "potentially symmetrical" bridged-T as shown in Fig. 1(d), we obtain the same values for  $\omega_z$  and  $q_z$  as in (3), but for the pole Q,  $\hat{q}$  we obtain:

$$\hat{q}_{\text{Bridged-T}} = \rho / (1 + 2\rho) \Big|_{\rho \rightarrow \infty} = 0.5. \quad (4)$$

It is apparent from (4) that by increasing the impedance-scaling factor  $\rho$  the passive pole Q,  $\hat{q}$  is increased towards 0.5. In what follows, we shall demonstrate the reduction of sensitivity when increasing the  $\rho$ , in the class-3 and class-4 filters, separately.

## III. CLASS-3 FILTERS—NEGATIVE FEEDBACK

Consider a common 2<sup>nd</sup>-order class-3 active-RC filter section with BP characteristic shown in Fig. 2. It is known as the Deliyannis section [3], and is suitable for realization of medium-Q values ( $2 < q_p < 20$ ) [5]. The circuit in Fig. 2 has the same bridged-T network as in Fig. 1, providing FRN characteristic in the negative feedback loop (from node 3 to 2), while in the signal forward path (node 1 to 2) it has a BP RC ladder network. Because of the latter, its overall BP transfer function is given by:

$$T(s) = \frac{V_{out}}{V_{in}} = \frac{K a_1 s}{s^2 + a_1 s + a_0} = \frac{K \cdot (\omega_p / q_p) \cdot s}{s^2 + (\omega_p / q_p) \cdot s + \omega_p^2}, \quad (5)$$

where the pole frequency,  $\omega_p$  and pole Q,  $q_p$  (or the transfer function coefficients  $a_1=\omega_p/q_p$  and  $a_0=\omega_p^2$ ) are given by:

$$K = q_p \beta \sqrt{\frac{R_2 C_2}{R_1 C_1}}; \quad \omega_p = \frac{1}{\sqrt{R_1 R_2 C_1 C_2}}; \quad q_p = \frac{\sqrt{R_1 R_2 C_1 C_2}}{R_1 (C_1 + C_2) + R_2 C_2 (1 - \beta)}, \quad (6)$$

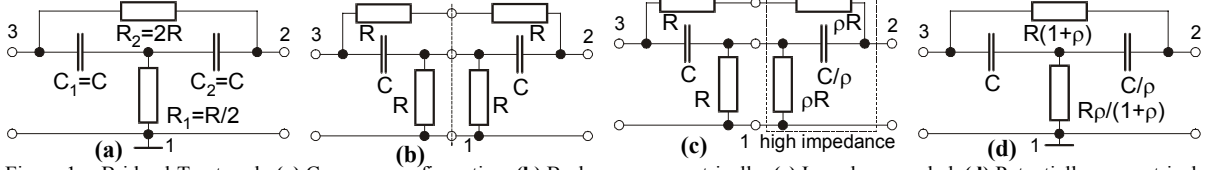


Figure 1. Bridged-T network. (a) Common configuration. (b) Broken up symmetrically. (c) Impedance scaled. (d) Potentially symmetrical.

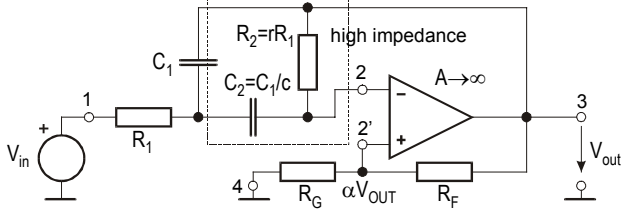


Figure 2. 2<sup>nd</sup>-order BP active-RC filter (Deliyannis SAB section) with general scaling factors  $r$  and  $c$  as in [6].

and where  $\bar{\beta} = 1 + R_G/R_F$  (7) represents a positive feedback gain, realized by resistors  $R_G$  and  $R_F$ . A positive feedback in the circuit, is represented by a direct measure  $\alpha = R_G/(R_F + R_G)$ ;  $0 < \alpha < 1$  in Fig. 2 (node 3 to 2').

The complementary transformation between class-3 network with additional positive feedback and class-4 network is demonstrated in [3] (p. 167) and [7]. In [7] it was shown that a 2<sup>nd</sup>-order class-4 high-pass (HP) filter and class-3 BP filter as shown in Fig. 2, are related by the complementary transformation, and the low-sensitivity design of one will produce the low-sensitivity design of the other filter; their coefficient-to-component sensitivities also have the same form. It is more practical to use  $\bar{\beta} = (1 - \alpha)^{-1}$ ;  $0 < \bar{\beta} < \infty$  instead of  $\alpha$ , because the equations for pole parameters as functions of components (6) are then identical for both filters [6]. Note that the class-4 (HP) circuit has the gain  $\beta = 1 + R_F/R_G$  instead of gain  $\bar{\beta}$ ; the gains are related by  $1/\bar{\beta} + 1/\beta = 1$  [7]. Sensitivity of coefficient  $a_0$  to all components is  $-1$ , thus only the sensitivities of  $a_1$  are presented in the first column of Table 1. Note that all sensitivities in Table 1 are proportional to pole  $Q$ ,  $q_p$ .

*Example:* Consider BP and BR filters having 1kHz center frequency and pass-band range of 200Hz. To obtain this selectivity we need the pole  $Q$  factor of  $q_p = \omega_0/B = 5$ , and the active-RC filter realization. Magnitudes of the transfer function characteristics are shown in Fig. 3.

One segment of the sensitivity analysis (given in [6]) of the SAB filter in Fig. 2 will be summarized here. Filters with various resistance ( $r$ ) and capacitance ( $c$ ) ratios are presented Table 2. In the last two columns the  $Q$ -values of the bridged-T network calculated

TABLE I. SENSITIVITY OF COEFFICIENT  $a_1$  TO COMPONENT VARIATIONS IN A 2<sup>ND</sup>-ORDER BAND-PASS FILTER.

$x$	$-(1/q_p) \cdot S_x^{a_1}$		
$R_1$	$-\sqrt{\frac{R_2 C_2}{R_1 C_1}} \cdot (\bar{\beta} - 1)$	$-\frac{1+p}{p} (\bar{\beta} - 1)$	-0.8
$R_2$	$\sqrt{\frac{R_1 C_1}{R_2 C_2}} + \sqrt{\frac{R_1 C_2}{R_2 C_1}}$	1	1
$C_1$	$\sqrt{\frac{R_1 C_2}{R_2 C_1}} - \sqrt{\frac{R_2 C_2}{R_1 C_1}} (\bar{\beta} - 1)$	$(1+1/p)(1-\bar{\beta}) + 1/(1+p)$	$-0.8 + \frac{1}{1+p}$
$C_2$	$\sqrt{\frac{R_1 C_1}{R_2 C_2}}$	$\frac{p}{1+p}$	$\frac{p}{1+p}$

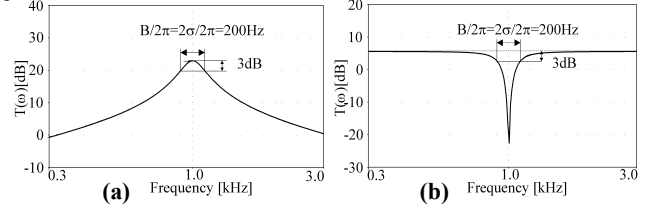


Figure 3. The 2<sup>nd</sup>-order filter transfer function magnitude with  $f_0=1$ kHz,  $q_p=5$ . (a) Band-pass. (b) Band-rejection.

from (2) are given. Corresponding Monte Carlo (MC) runs with 1% Gaussian distribution, zero-mean resistors and capacitors were carried out and presented in Fig. 4. It is obvious, for the reasons given in [6] that filter no. 4 with equal capacitors and impedance scaled resistors has min. sensitivity.

When used in the “infinite-gain” mode the closed-loop poles in (5) of the class-3 network coincide with the open-loop zeros of bridged-T in (1). By applying an additional positive feedback the closed-loop poles move closer to the  $j\omega$ -axis, starting from the zeros of the bridged-T. Therefore, from (2) and (6) we have the relationship for pole  $Q$ ,  $q_p$ , which is given by ([3] p. 161):

$$q_p = \hat{q} [1 - \bar{\beta} (1 - \hat{q}/q_z)]^{-1}; \quad \hat{q} < q_z \leq q_p. \quad (8)$$

From (8) obviously, the pole  $Q$ ,  $q_p$  has the form:

$$q_p = \kappa_1 \cdot (\kappa_2 - \bar{\beta} \kappa_3)^{-1}, \quad (9)$$

where  $\kappa_1$ ,  $\kappa_2$ , and  $\kappa_3$  depend only on the passive  $RC$  network [2]. Calculating the relative sensitivity of the pole  $Q$ ,  $q_p$  to the variations of positive feedback gain  $\bar{\beta}$ , we obtain [1][2]:

$$S_{\bar{\beta}}^{q_p} = -S_{\bar{\beta}}^{a_1} = q_p / \hat{q} - 1. \quad (10)$$

TABLE II. COMPONENT VALUES OF 2<sup>ND</sup>-ORDER BP FILTERS WITH VARIOUS SCALING FACTORS (RESISTORS IN [KΩ], CAPACITORS IN [NF]).

No.	$r$	$c$	$R_1$	$C_1$	$R_2$	$C_2$	$\bar{\beta}$	$\hat{q}$	$q_z$
1)	1	1	15.9	10	15.9	10	2.8	0.33	0.5
2)	4	4	15.9	10	63.6	2.5	2.05	0.44	0.8
3)	1	4	31.8	10	31.8	2.5	5.6	0.33	0.4
4)	4	1	7.96	10	31.8	10	1.4	0.33	1.0

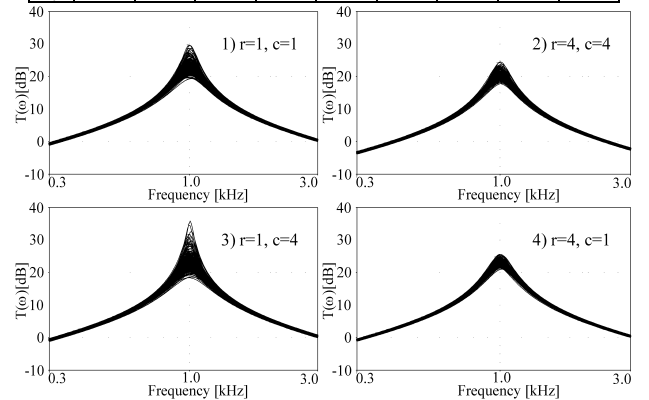


Figure 4. Monte Carlo runs of impedance-tapered 2<sup>nd</sup>-order BP filters given in Table 2.

Note that pole Q,  $q_p$  sensitivity to gain variation is reduced as the value of the passive pole Q,  $\hat{q}$  increases. Will the decrease of sensitivity in (10) really reduce the sensitivity to the tolerances of the two positive-feedback resistors  $R_G$  and  $R_F$ ? The relative variation of pole Q,  $q_p$ , due to variations of resistors  $R_F$  and  $R_G$ , is given by:

$$\frac{\Delta q_p}{q_p} = S_{\bar{\beta}}^{q_p} \cdot \frac{\Delta \bar{\beta}}{\bar{\beta}}; \text{ where } \frac{\Delta \bar{\beta}}{\bar{\beta}} = S_{R_G}^{\bar{\beta}} \frac{\Delta R_G}{R_G} + S_{R_F}^{\bar{\beta}} \frac{\Delta R_F}{R_F}. \quad (11)$$

From (7) we can readily calculate (in class-3) the sensitivity of the gain  $\bar{\beta}$  to the feedback resistors  $R_F$  and  $R_G$ :

$$S_{R_G}^{\bar{\beta}} = -S_{R_F}^{\bar{\beta}} = 1 - 1/\bar{\beta}. \quad (12)$$

Expressing  $\bar{\beta}$  from (8), and substituting it into (12) and with (11) we have:

$$S_{R_G}^{q_p} = -S_{R_F}^{q_p} = S_{\bar{\beta}}^{q_p} \cdot S_{R_G}^{\bar{\beta}} = q_p/q_z - 1. \quad (13)$$

Note that the  $q_p$  sensitivities to resistors  $R_G$  and  $R_F$  in (13) are independent of passive pole Q,  $\hat{q}$  value (the term  $q_p/\hat{q}-1$  has cancelled out). Furthermore, the sensitivities in (13) are inversely proportional to the  $q_z$ . Thus to reduce it, we should use large  $q_z$  values (note that filter no. 4 with min. sensitivity has largest  $q_z$  in Table 2). From (2) it is obvious that for large  $q_z$  we must let  $C_1=C_2$  and  $R_2/R_1=4q_z^2$  (large resistor spread is required). It is known [1] that by bridged-T  $\hat{q}$  and  $q_z$  are not independent. They cannot reach their respective max. values at the same time; the larger  $q_z$  is selected, the smaller  $\hat{q}$  becomes. If we want to reduce the sensitivity of the circuit in Fig. 2 as much as possible, we reach the case when there is no positive feedback at all ( $q_p=q_z$ ,  $\bar{\beta}=1$ ,  $R_2/R_1=4q_p^2$ !). The ‘‘medium-Q’’ circuit in Fig. 2, then simplifies into ‘‘low-Q’’ circuit ( $\bar{\beta}=1$ ) as defined in [5]. It provides a good solution when pole Q is smaller than 2. ‘‘Low-Q’’ circuit is not suitable for larger pole Q realizations because its component spread and the gain-sensitivity-product (GSP)<sup>†</sup> are both proportional to  $q_p^2$ . By ‘‘medium-Q’’ filter ( $\bar{\beta} > 1$ ) the GSP is proportional to  $q_p$  and the component spread is not so critical [5].

In the new design instead of general scaling factors  $r$  and  $c$  in Fig. 2 we introduce scaling factor  $\rho$  as in Fig. 1(d) to calculate elements of potentially symmetrical bridged-T, using:

$$R_1=\rho/(1+\rho)R; C_1=C; R_2=(1+\rho)R; C_2=C/\rho. \quad (14)$$

With (14) we obtain the sensitivity relations given in the second column of Table 1. With  $\bar{\beta}$  from (8), and with (3), (4) and (14) for the potentially symmetrical bridged-T we have:

$$\bar{\beta}=1+(1-q_p^{-1}) \cdot \rho/(1+\rho) \quad (15)$$

Substituting (15) into the second column of Table 1, we obtain the sensitivities in its last column (in our example of  $q_p=5$  the term  $(1-q_p^{-1})$  in (15) equals 0.8). Obviously, only the sensitivities to  $C_1$  and  $C_2$  are dependent on the factor  $\rho$  and they worsen as  $\rho$  (i.e. the value of  $\hat{q}$ ) increase. From (13) we have  $S_{R_G, R_F}^{q_p} = \pm 4$ .

To double-check the above conclusions we designed the filter in Fig. 2, with two values of  $\rho$ . The component values of the resulting filters are in Table 3, and MC runs are in Fig. 5. It appears that as  $\rho$  increases, the sensitivities in Fig. 5 are getting slightly worse. Thus, lower sensitivity of the two has the filter no. 1) which has  $\rho=1$  and a symmetrical bridged-T network. Incidentally it is the identical filter to the filter no. 4) with min. sensitivity in Table 2 (and in [6]). Obviously, here tapering with potentially symmetrical bridged-T does not help. In [5] are given design procedures for min.-GSP

<sup>†</sup> The GSP gives a measure of a filter’s magnitude sensitivity to the open-loop gain ( $A$ ) variation of the active component [5].

TABLE III. COMPONENT VALUES OF 2<sup>ND</sup>-ORDER BP FILTERS WITH POTENTIALLY SYMMETRICAL BRIDGED-T.

No.	$\rho$	$R_1$	$C_1$	$R_2$	$C_2$	$\bar{\beta}$	$\hat{q}$	$q_z$
1)	1	7.96	10	31.8	10	1.4	0.33	1.0
2)	4	12.7	10	79.6	2.5	1.64	0.44	1.0

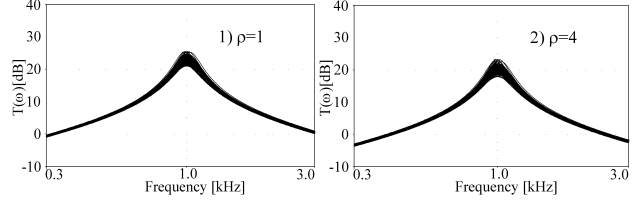


Figure 5. Monte Carlo runs of impedance-tapered 2<sup>nd</sup>-order BP filters given in Table 3.

biquads. In most of the circuits in [5], one additional degree of freedom is available, which permits to choose the values and ratios of two (or three) components. The optimum trade-off in the Deliyannis circuit between  $q_z$  (neg. feedback) and  $\bar{\beta}$  (pos. feedback) is to choose resistor ratio  $R_2/R_1 > 1$  (by which the  $q_z$  is increased) thus reducing passive sensitivity, and to calculate capacitor ratio  $C_1/C_2$  for min. GSP (from [5] p. 54), which provides circuit with reduced active sensitivity, as well.

#### IV. CLASS-4 FILTERS—POSITIVE FEEDBACK

As a representative example consider a 2<sup>nd</sup>-order class-4 BR (or notch) filter shown in Fig. 6, which is known as ‘‘Split-Feedback FRN’’ (SF-FRN) [3]. It is suitable for medium-Q realizations [5]. It has a potentially symmetrical ‘‘Twin-T’’ circuit in the positive feedback loop (inside rectangle) [1]. In [3] (pp. 224-229) the ‘‘Standard’’ FRN (ST-FRN) is presented, which has feedback on both  $R_3$  and  $C_3$  legs (switch  $S_1$  is in the position ‘‘ST-FRN’’). To make possible the realization of finite pole-Q,  $q_p$  ST-FRN needs a ‘‘loading network’’ of twin-T as in Fig. 6 [3]. In what follows we shall concentrate on SF-FRN because it is much simpler. It can readily be shown that the design techniques and results obtained for SF-FRN can be applied for the design of low-sensitivity ST-FRN, and for all-pass (AP) networks in [3], as well.

The transfer function is given by:

$$T(s) = \frac{V_{out}}{V_{in}} = K \frac{s^2 + \omega_z^2}{s^2 + (\omega_p/q_p) \cdot s + \omega_p^2}. \quad (16)$$

The parameters in (16) are given by [3]:

$$K = \beta; \omega_p^2 = \omega_z^2 = \frac{1}{R_1 R_2 C_3 C_3} = \frac{1}{C_1 C_2 R_3 R_3}; q_p = \frac{R_2 C_1}{C_2 R_s + C_3 R_1 (1 - \beta)}, \quad (17)$$

where

$$\beta = 1 + R_F/R_G \quad (18)$$

is the positive feedback gain in the class-4 circuits and

$$C_s = C_1 C_2 / (C_1 + C_2), R_s = R_1 + R_2, R_p = R_1 R_2 / (R_1 + R_2), C_p = C_1 + C_2. \quad (19)$$

Note also that  $q_z = \infty$ , if the ‘‘balance condition’’ for the Twin-T network holds, and it is given by [1]:

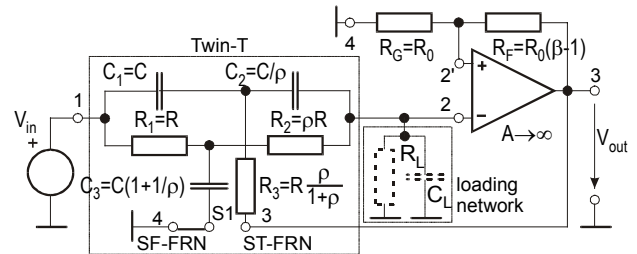


Figure 6. 2<sup>nd</sup>-order BR active-RC filter of class-4: ‘‘Split Feedback Twin-T’’ section as in [3].

$$R_3/C_3 = R_p/C_p. \quad (20)$$

Introducing the impedance-scaling factor  $\rho$ , to obtain the potentially symmetrical twin-T, into (17), i.e.

$$R_1=R; C_1=C; R_2=\rho R; C_2=C/\rho; R_3=\rho R/(1+\rho); C_3=C(\rho+1)/\rho \quad (21)$$

we satisfy (20) and obtain the following simple relations:

$$\omega_p = \omega_z = \frac{1}{RC}; q_z = \infty; q_p = \hat{q} \cdot \frac{1}{1-\beta/2}; \hat{q}_{\text{Twin-T}} = \frac{1}{2} \cdot \frac{\rho}{1+\rho}. \quad (22)$$

Note that the impedance-scaling factor  $\rho$  exists only in equation for calculating  $\hat{q}$ . If we increase  $\rho \rightarrow \infty$ , the passive pole Q,  $\hat{q} \rightarrow 0.5$ .

The process of deriving a potentially symmetrical twin-T network [1][4] is identical as for the bridged-T network presented above. It is actually a third-order network but because of (20) the negative real pole and zero are cancelled out. Therefore the twin-T passive network provides a 2<sup>nd</sup>-order transfer function of the form given by:

$$\hat{i}_{12}(s) = \frac{V_2}{V_1} = k_{12} \frac{s^2 + \omega_z^2}{s^2 + (\omega_p/\hat{q}) \cdot s + \omega_0^2}, \quad (23)$$

For the symmetrical case with  $\rho=1$ , the parameters in (23) take on the values in (22), and  $k_{12}=1$ ,  $\hat{q}=1/4$ .

It is well known [1] that the network in the positive feedback loop of the class-4 circuit has a BP characteristic of the form:

$$\hat{i}_{32}(s) = \frac{V_2}{V_3} = 1 - \hat{i}_{12}(s) = \frac{\omega_k \cdot s}{s^2 + (\omega_0/\hat{q}) \cdot s + \omega_0^2} \quad (24)$$

and therefore the pole parameters in (16) have the form:

$$\omega_p = \omega_0; q_p = \hat{q} \cdot (1 - i \cdot \beta \hat{q} \omega_k / \omega_0)^{-1}, \quad (25)$$

where  $i=1$  or  $0.5$  [we have  $\beta$  or  $\beta/2$  in (25)] for ST- and SF-FRN, respectively. Note that  $\omega_p$  is independent of  $\beta$ , and  $q_p$  has the form presented in (9). Thus the relative sensitivity of the pole Q,  $q_p$  to the variations of gain  $\beta$ ,  $S_{\beta}^{q_p}$  has the form given by (10) above (with  $\beta$  instead  $\bar{\beta}$ ). Furthermore, from (18) the sensitivity of the gain  $\beta$  to the  $R_F$  and  $R_G$  follows (in class-4) and it is given by:

$$S_{R_F}^{\beta} = -S_{R_G}^{\beta} = 1 - 1/\beta. \quad (26)$$

With a potentially symmetrical twin-T network ST-FRN has the gain  $2\beta$  in (17) and (22) instead of  $\beta$ , and the other expressions in (17) apply. Generally, from (25) and (26) we calculate the relative sensitivity of the pole Q,  $q_p$  to  $R_F$  and  $R_G$ , and it is given by:

$$S_{R_F}^{q_p} = -S_{R_G}^{q_p} = S_{\beta}^{q_p} \cdot S_{R_F}^{\beta} = q_p \cdot (\hat{q}^{-1} - i \cdot \omega_k / \omega_0) - 1, \quad (27)$$

where  $i=1$  or  $0.5$ . From (22) and (25) and with  $i=0.5$  we have  $\omega_k/\omega_0=1/\hat{q}$ ; for the special case SF-FRN we can rewrite (27) into:

$$S_{R_F}^{q_p} = -S_{R_G}^{q_p} = 0.5 \cdot q_p / \hat{q} - 1. \quad (28)$$

Unlike in (13), note that the  $q_p$  sensitivities to resistors  $R_G$  and  $R_F$  in (28) are inversely proportional to the passive pole Q,  $\hat{q}$  value. The reason for this lies in the different techniques of obtaining pole Q values for class-3 and class-4 networks given above. Thus the decrease of sensitivity in (10) will reduce the sensitivity to the feedback-resistor tolerances in the case of class-4 networks. Both sensitivities in (13) and (28) are proportional to the pole Q,  $q_p$  which is the characteristic of "medium-Q" filters [3].

Furthermore, it has been investigated using the program "Mathematica", and the results of this investigation show that increasing the scaling factor  $\rho$  will also decrease the coefficient  $a_1$  sensitivities to the passive components  $R_i$  and  $C_i$  ( $i=1,2,3$ ).

To double-check the above conclusions we designed the filter in Fig. 6, with two values of  $\rho$  for the BR example above. The resulting filters are in Table 4 and corresponding MC runs are in Fig. 7. Note that in the second column in Fig. 7 we perform only variation of feedback resistors  $R_G$  and  $R_F$ .

TABLE IV. COMPONENT VALUES OF 2<sup>ND</sup>-ORDER BR FILTERS WITH POTENTIALLY SYMMETRICAL TWIN-T.

No.	$\rho$	$R_1$	$R_2$	$R_3$	$C_1$	$C_2$	$C_3$	$\beta$	$\hat{q}$
1)	1	15.9	15.9	7.95	10	10	20	1.90	0.25
2)	4	15.9	63.6	12.7	10	2.5	12.5	1.84	0.40

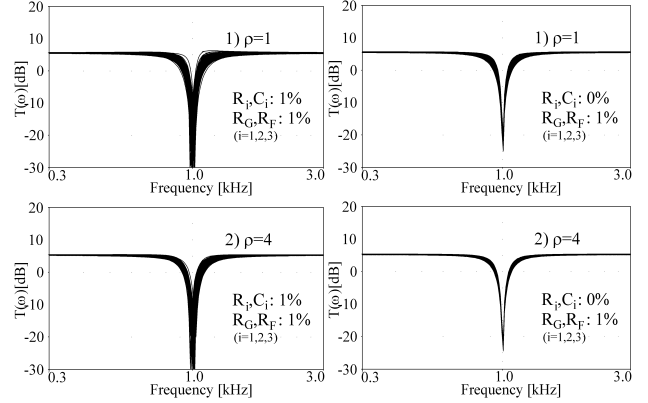


Figure 7. Monte Carlo runs of impedance-tapered 2<sup>nd</sup>-order BR filters given in Table 4.

Clearly, as  $\rho$  increases, the sensitivities in Fig. 7 are improving. We can conclude that: *we should enlarge the design factor  $\rho$  in (22) in the design procedure for class-4 filters.*

## V. CONCLUSIONS

In this paper we described the design procedure of the active-RC filters, which have a symmetrical bridged-T and twin-T networks in negative (class-3) and positive (class-4) feedback loops. The process of deriving "potentially symmetrical" networks by increasing the impedance of one half of those symmetrical networks  $\rho$  times, has been described in [1][4]. Choosing a scaling factor  $\rho > 1$  the passive pole Q factor,  $\hat{q}$  of both networks is increased. It is shown that for class-4 filters (Twin-T BR filter) the passive sensitivity can be reduced choosing larger  $\rho$  (and consequently larger  $\hat{q}$ ). For the class-3 filters (Deliyannis BP section) the passive sensitivity can only be reduced by increasing the  $q_z$  value of bridged-T, even though the  $\hat{q}$  is thereby decreased. For the latter it is optimal to choose resistive impedance tapering with either equal capacitors or capacitors selected for GSP-minimization [5].

## REFERENCES

- [1] G. S. Moschytz, Linear integrated networks: fundamentals, NY (Bell Labs Series): V. Nostrad Reinhold Co., 1974.
- [2] G. S. Moschytz, "Low-sensitivity, low-power, active-RC allpole filters using impedance tapering," IEEE Trans. on CAS—II: vol. CAS-46, no. 8, pp. 1009-1026, Aug. 1999.
- [3] G. S. Moschytz, Linear integrated networks: design, New York (Bell Labs Series): Van Nostrad Reinhold Co., 1975.
- [4] C. Toumazou, G. S. Moschytz and B. Gilbert, Trade-offs in analog circuit design: the designer's companion, Kluwer Academic Publishers, September 2002.
- [5] G. S. Moschytz and P. Horn, Active Filter Design Handbook, Chichester, U.K.: Wiley 1981.
- [6] D. Jurišić, G.S. Moschytz and N. Mijat, "Low-sensitivity SAB band-pass active-RC filter using impedance tapering," in Proc. ISCAS'01, (Sydney, Australia), vol. 1, pp. 160-163, May 6-9, 2001.
- [7] G. S. Moschytz and P. Horn, "Optimizing two commonly used active-filter building blocks using the complementary transformation," Electronic Circuits and Systems, vol. 1, no. 4, pp. 125-132, July 1977.