A NOVEL METHOD FOR THE CLOSED-FORM ANALYSIS AND DESIGN OF A 4TH-ORDER SINGLE-AMPLIFIER FILTER

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ABSTRACT

In this paper we present a new analytical design procedure for arbitrary-order, single-amplifier class-4 active-*RC* allpole filters. It is already known that only 2nd- and 3rd-order filters can be designed analytically using classical design methods [1] and the equations of higher order still defy a closed-form solution. The new design method parses the ladder network into consecutive L-sections, and makes use of their subsequent transfer functions that can be calculated iteratively. This will be demonstrated in an example of the closed-form analysis and design of a special case of a 4th-order unity-gain LP filter. The Butterworth and Chebyshev approximations with pass-band ripple 0.1-0.2[dB] are considered. The resulting expressions provide insight into the feasibility of realizing a 4th-order filter. The procedure used for the analysis is mainly not only of academic interest, but may have practical significance for special problems.

1. INTRODUCTION

In [1] it was shown that the active-*RC* Sallen and Key [2] (class 4) allpole filters of 2nd- and 3rd-order have an analytical solution. The realizability constraints are given in [3]. Unfortunately, the elements of the 4th- and higher-order filter cannot be calculated analytically e.g. by comparing the transfer function (TF) parameters with the parameters of the chosen filter circuit. In this paper we present a new analysis and design concept, by which the problems arising in the design of higher-than-third order filters can be solved. In doing so we use the program MATHEMATICA.

In what follows, a 4th-order low-pass (LP) active-*RC* allpole filter with single operational amplifier (opamp) is designed. It will be shown that an analytical solution is possible for the special case of a unity-gain filter (β=1). As an extension of the procedure given in [4], we present the design equations and the realizability constraints, which also have an analytical form.

2. FOURTH-ORDER ALLPOLE FILTERS

Consider the 4th-order LP filter shown in Figure 1.

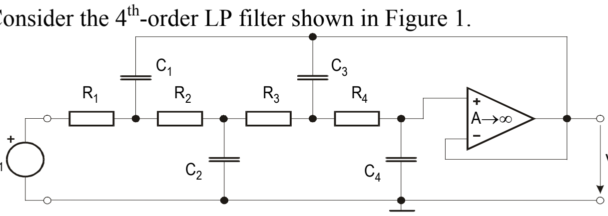

Figure 1. Fourth-order LP filter with single opamp and unity gain.

It is a low-power circuit, in that it uses only one opamp. The voltage transfer function $T(s)$ of this section in terms of the coefficients a_i; i=0,…, 3 is given by:

$$T(s)=\frac{V_2}{V_1}=\frac{\beta a_0}{s^4+a_3s^3+a_2s^2+a_1s+a_0}\,, \tag{1}$$

and in terms of pole Q-factors q_{pi}, and pole frequencies ω_{pi}; i=1, 2, is given by:

$$T(s)=\frac{\beta\omega_{p1}^2\omega_{p2}^2}{\left[s^2+(\omega_{p1}/q_{p1})s+\omega_{p1}^2\right]\left[s^2+(\omega_{p2}/q_{p2})s+\omega_{p2}^2\right]}\,. \tag{2}$$

The coefficients a_i; i=0,…3 as a function of components of the circuit are given in [5]. In the case of the filter shown in Figure 1 the gain β is fixed at unity (β=1), and it is realized by an opamp interconnected as a voltage follower. Note that the poles and zeros of the unity-gain filter are exclusively determined by the passive-*RC* network.

2.1. n^{th}-order Passive-*RC* Ladder Network

Consider a passive-*RC* ladder network in the signal forward path, i.e. from the driving source to the opamp input in Figure 1 when the gain β=0 (the opamp is turned off). The voltage transfer function of the passive-*RC* ladder-network, which has impedances Z_i in the series branches and admittances Y_i in the shunt branches, is given by:

$$H=V_n/V_0=(Z_1—Y_n)^{-1}\,, \tag{3}$$

where for a 4th-order LP case Z_k=R_k, Y_k=sC_k; k=1,…,n; n=4. The denominator (Z_1—Y_n) in (3) is a well-known determinant, so-called continuant, which can readily be solved using recursive formulae [6][7]. We can represent the continuant (Z_1—Y_n) in the form of a sum of polynomials D_k plus constant one. The D_k are k^{th}-order polynomials in variable s, which can also be calculated using recursive formulae. Every polynomial D_k has a single root at the origin, and k-1 roots that are real and negative. Equation (3) takes on the form:

$$H=V_n/V_0=\left(1+\sum\nolimits_{k=1}^{n}D_k\right)^{-1}\,, \tag{4}$$

$$D_k=\frac{C_kR_k}{C_{k-1}R_{k-1}}\left[D_{k-1}\left(1+sC_{k-1}R_{k-1}+\frac{R_{k-1}}{R_k}\right)-\frac{C_{k-1}}{C_{k-2}}D_{k-2}\right], \tag{5}$$

where D_1=sC_1R_1, D_0=0, k=2,…,n. The equation (5) can also be rewritten in the form:

$$D_k=sC_kR_k\left(\sum\nolimits_{i=1}^{k-1}D_i+1\right)+C_kC_{k-1}^{-1}D_{k-1}\,. \tag{6}$$

The coefficients of the polynomials in (6) are given by:

$$D_k=\sum_{i=1}^{k}b_{ik}s^i\ ;\ \ b_{1k}=\frac{C_k}{C_{k-1}}b_{1,k-1}+C_kR_k\ ;\ k=1,\ldots,n; \tag{7}$$

$$b_{ik}=\frac{C_k}{C_{k-1}}b_{i,k-1}+C_kR_k\sum_{r=1}^{k-1}b_{i-1,k-r}\ ;\ b_{ik}=0\ ;\ i=2,\ldots,k;\ i>k.$$

2.2. n^{th}-order *RC* Ladder Network with an Active Element

If we insert the voltage-controlled-voltage source, controlled by the output voltage V_n, into the r^{th} shunt branch, and with (4), the resulting transfer function is given by:

$$F = \frac{V_{out}}{V_0} = \beta \left[1 + \sum_{k=1}^{n} (1 - \delta_r \beta) D_k \right]^{-1}; \quad V_{out} = \beta V_n \,, \tag{8}$$

with δ_r=1 for the r^{th} branch, where the controlled source is inserted, and δ_r=0 for the other branches. We insert a source into every odd or even shunt branch of the ladder network to build filters of even and odd order n, respectively.

In the special case when β=1, some of the polynomials D_k in the transfer function (8) vanish, and we have the following voltage transfer functions for n:

$$\underline{\text{even:}}\ F_n = \left[1 + \sum_{k=1}^{n/2} D_{2k} \right]^{-1}; \quad \underline{\text{odd:}}\ F_n = \left[1 + \sum_{k=1}^{(n+1)/2} D_{2k-1} \right]^{-1}. \tag{9}$$

3. DESIGNING 4TH-ORDER UNITY-GAIN LP FILTER

In what follows, we shall use the fact that some of the polynomials in the transfer function (8) disappear. Therefore we can represent the 4th-order transfer function as the sum of 2nd- and 4th-order polynomials D_2 and D_4.

As a starting point for our design method we use the well-known mathematical hypothesis [4]: "generally, any polynomial of n^{th}-order with complex roots can always be represented as a sum of polynomials with real and negative roots and one root in the origin, plus some constant." That sum contains only odd polynomials if the order n is odd, and only even polynomials if the order n is even. That property leads us to the possibility of identification of those polynomials with the polynomials D_k, as presented in the previous paragraph, and it is the appropriate formulation for our filter design.

We start our design procedure from parameters given in the form of pole Q-factors q_{pi}, and frequencies ω_{pi}; i=1, 2, rather than coefficients a_i; i=0,…, 3. From (9) follows the 4th-order filter transfer function F_4, which can be equated to the desired filter specifications:

$$F_4^{-1} = D_4 + D_2 + 1 = \prod_{i=1,2} [s^2 / \omega_{pi}^2 + s / (q_{pi} \omega_{pi}) + 1] \,. \tag{10}$$

By introducing the parameter m, which can be freely chosen inside the boundaries $0<m<1$, we have the following polynomials:

$$D_4 = b_4 s (s + b_3)(s + b_2)(s + b_1); \ D_2 = c_2 s (s + c_1) \,, \tag{11}$$

where the roots are given by:

$$b_4 = \frac{1}{\omega_{p1}\omega_{p2}}; \ b_3 = \frac{\omega_{p1}}{q_{p1}}; \ b_2 = m\frac{\omega_{p2}}{q_{p2}}; \ b_1 = (1-m)\frac{\omega_{p2}}{q_{p2}};$$

$$c_2 = \frac{1}{\omega_{p1}^2 \omega_{p2}^2} \left[\omega_{p1}^2 + \omega_{p2}^2 - m(1-m)\frac{\omega_{p2}^2}{q_{p2}^2} \right]; \tag{12}$$

$$c_1 = \frac{1}{\omega_{p1}\omega_{p2} c_2} \left[\left(\frac{\omega_{p2}}{q_{p1}} + \frac{\omega_{p1}}{q_{p2}} \right) - m(1-m)\frac{1}{q_{p1}}\frac{\omega_{p2}}{q_{p2}^2} \right].$$

From (12) it is obvious that the values of m and (m-1) produce the same roots of the polynomials D_2 and D_4. Because of this symmetry it is sufficient to investigate the influence of the constant m in the interval $0<m<0.5$.

Consequently we can write the polynomials:

$$D_4 = b_{44}s^4 + b_{34}s^3 + b_{24}s^2 + b_{14}s; \ D_2 = b_{22}s^2 + b_{12}s \,, \tag{13}$$

with the coefficients given by:

$$b_{44} = \frac{1}{\omega_{p1}^2\omega_{p2}^2}; \qquad b_{34} = \frac{1}{\omega_{p1}^2\omega_{p2}^2}\left(\frac{\omega_{p1}}{q_{p1}} + \frac{\omega_{p2}}{q_{p2}} \right); \tag{14}$$

$$b_{24} = \frac{1}{\omega_{p1}^2\omega_{p2}^2}\left(\frac{\omega_{p1}\omega_{p2}}{q_{p1}q_{p2}} + m(1-m)\frac{\omega_{p2}^2}{q_{p2}^2} \right); \ b_{14} = \frac{1}{\omega_{p1}^2\omega_{p2}^2}\frac{\omega_{p1}}{q_{p1}} m(1-m)\frac{\omega_{p2}^2}{q_{p2}^2};$$

$$b_{22} = \frac{1}{\omega_{p1}^2\omega_{p2}^2}\left[\omega_{p1}^2 + \omega_{p2}^2 - m(1-m)\frac{\omega_{p2}^2}{q_{p2}^2} \right]; \tag{14}$$

$$b_{12} = \frac{1}{\omega_{p1}^2\omega_{p2}^2}\left[\omega_{p1}\omega_{p2}\left(\frac{\omega_{p2}}{q_{p1}} + \frac{\omega_{p1}}{q_{p2}} \right) - m(1-m)\frac{\omega_{p1}}{q_{p1}}\frac{\omega_{p2}^2}{q_{p2}^2} \right].$$

It is obvious that using the parameter m as a free parameter, we can generate an infinite number of polynomials D_2 and D_4 which will satisfy the desired transfer function.

We choose m and calculate the polynomials D_2 and D_4 using (14). From (7) the system of ten equations with twelve unknowns for the coefficients of the polynomials D_1 to D_4 follows. Therefore, two variables have to be chosen. If we introduce the following capacitor ratios:

$$X_1 = C_2 / C_1; \quad X_2 = C_3 / C_2; \quad X_3 = C_4 / C_3 \,, \tag{15}$$

we reduce the number of unknowns by one. Furthermore, we choose the value for the unknown b_{11}, and after elimination of R_1, R_2, R_3 and R_4, we have a system with six equations and six unknowns: X_1, X_2, X_3, b_{13}, b_{23} and b_{33}. The system is given by:

$$(i)\ b_{12} = X_1 b_{11} + \frac{b_{22}}{b_{11}}; \ (ii)\ b_{13} = X_2 b_{12} + \frac{b_{33}}{b_{22}};$$

$$(iii)\ b_{23} = X_2 b_{22} + \frac{b_{33}}{b_{22}}(b_{12} + b_{11}); \ (iv)\ b_{14} = X_3 b_{13} + \frac{b_{44}}{b_{33}} \tag{16}$$

$$(v)\ b_{24} = X_3 b_{23} + \frac{b_{44}}{b_{33}}(b_{13} + b_{12} + b_{11}); \ (vi)\ b_{34} = X_3 b_{33} + \frac{b_{44}}{b_{33}}(b_{23} + b_{22}).$$

From the first equation we have a value for X_1, i.e.

$$X_1 = (b_{11}b_{12} - b_{22}) / b_{11}^2 \tag{17a}$$

Solving the other equations for X_2 and X_3 we obtain:

$$X_2 = \frac{J_1^2}{(J_2 b_{12} - J_3 b_{14})J_1 - J_2 J_3 b_{44} / b_{22}}, \ X_3 = \frac{J_2}{X_2 J_3}, \tag{17b, 17c}$$

where we used the abbreviations:

$$J_1 = E_1 E_3 - b_{12} E_2, \quad J_2 = E_2 + E_3, \quad J_3 = E_1 + b_{12},$$

$$E_1 = \frac{b_{11}}{b_{12}} - b_{12} - b_{11}, \ E_2 = \frac{b_{24}}{b_{12}} - \frac{b_{14}}{b_{12}}(b_{12} + b_{11}) - \frac{b_{44}}{b_{22}b_{12}}, \tag{18}$$

$$E_3 = b_{14} + b_{44}(b_{12} + b_{11}) / b_{22}^2 - b_{34} / b_{22} \,.$$

From the values of X_1, X_2, and X_3, we calculate the other unknown parameters in the system of equations (16), i.e. the coefficients b_{13}, b_{23} follow from (16) (ii) and (iii), while b_{33} from:

$$b_{33} = \frac{b_{44} X_2}{X_2 X_3 b_{12} - E_3} \,. \tag{19}$$

To calculate the remaining elements in the network we have to choose one component, e.g. C_1; the remaining components follow from: $R_1 = b_{11}/C_1$; $C_2 = X_1 C_1$; $C_3 = X_2 C_2$; $C_4 = X_3 C_3$; $R_2 = b_{22}/(C_2 b_{11})$; $R_3 = b_{33}/(C_3 b_{22})$; $R_4 = b_{44}/(C_4 b_{33})$.

The only free parameters to be chosen in our analytical procedure are the design coefficient m and the parameter b_{11}. Note that parameter $b_{11} = (\omega_0)^{-1}$, where $\omega_0 = (R_1 C_1)^{-1}$ represents the "design frequency" as defined in [1].

4. EXAMPLE

In what follows we construct a unity-gain 4th-oder LP filter. Consider the Butterworth and Chebyshev filter specifications having a pass-band ripple, R_p=0.1 and 0.2[dB] with the coefficients shown in Table 1. The corresponding magnitudes α(ω)[dB] with cut-off frequencies normalized to 1[rad/s] are shown in Figure 3(a).

Table 1. Butterworth* and Chebyshev Specifications Coefficients.

No.	Specif.	R_p [dB]	a_0 $\cdot 10^{22}$	a_1 $\cdot 10^{17}$	a_2 $\cdot 10^{11}$	a_3 $\cdot 10^{5}$	ω_{p1} $\cdot 10^{5}$	ω_{p2} $\cdot 10^{5}$	q_{p1}	q_{p2}
1)*	1.0/40dB; 50/250kHz	0.0	6.09	3.20	8.42	13.0	4.97	4.97	0.54	1.31
2)	0.1/47dB; 80/300kHz	0.1	5.29	2.57	6.64	9.07	3.97	5.80	0.62	2.18
3)	0.2/50dB; 80/300kHz	0.2	3.76	1.93	5.50	7.72	3.52	5.50	0.65	2.43

4.1. The Realizability Constraints

To investigate the realizability constraints, i.e. the permissible range of b_{11}, we consider the values X_1, X_2, X_3, b_{33} as a function of b_{11}, with m as a parameter. The values of the parameters X_1, X_2, X_3, and b_{33}, defined by equations (17) and (19), must be real and positive. From (17c) it is obvious that instead of X_3 it is more convenient to observe the ratio J_2/J_3. We start with the Butterworth coefficients given in the first line in Table 1, choose m=0.2, and present the functions X_1, X_2, J_2/J_3, and b_{33} vs. wide range of values b_{11} in Figure 2(a)-(d), respectively. Note that the functions under consideration are rational polynomial functions, which have poles and zeros.

To qualitatively represent the flow of the functions in Figure 2 vs. parameter b_{11}, we refer to Table 2, in which every row corresponds to one function. Both in Figure 2 and Table 2 the poles are denoted by "x" and zeros by "o". Between the poles and zeros considered, the functions take positive (denoted by "+") or negative ("-") values. Note that there exists an interval of b_{11} values, where all four functions are positive, and that area is grayed in Table 2. By increasing the value of parameter m, we obtain the case denoted by "B" in which the second zero and third pole of the function X_2 are exchanged. The remaining functions only shift their characteristic points, i.e. the poles and zeros, left or right. Two cases A and B correspond to values of parameter m=0.2, and 0.42, respectively. Table 2 has a property that characteristic points, which lie in the same column, have identical values (for case A and B separately).

Table 2. Polarity of Functions X_1, X_2, X_3, b_{33} vs. b_{11} (see Figure 2).

X_1	-					o	+	+				+					A
X_2	+	o	+	x	-			x	+			+	o	+	x	-	
J_2/J_3	-					x	+	+			o	-					
b_{33}	+	o	-	x	+	o	-	x	+			+	o	-	x	+	
X_2	+	o	+	x	-			x	+	x	-			o	-		B
J_2/J_3	-					x	+	+			+	o	-				

→ b_{11}

In what follows, we try to find the closed-form equations for calculations of zeros and poles of the functions shown in Figure 2. To ensure the positive value of X_1, from (17a) we have the first constraint given by:

$$b_{11} = R_1 C_1 > b_{22} / b_{12} = b_{X1z} \ . \tag{20}$$

Note that the curve X_1 in Figure 2(a), has only one zero b_{X1z}, and that X_1 is positive for the values b_{11}>b_{X1z}.

We proceed with the observation of the function X_2, shown in Figure 2(b). In order to find critical zeros and poles we equate the numerator of (17b) to zero, i.e. J_1=0, which results in the quadratic eq., with roots b_{X2z1}, b_{X2z2} (zeros). They are not particularly useful, thus we continue by equating the denominator of (17b) to zero:

$$(J_2 b_{12} - J_3 b_{14}) J_1 - J_2 J_3 b_{44} / b_{22} = 0 \ , \tag{21}$$

which results in a cubic equation, with roots b_{X2p1}, b_{X2p2}, b_{X2p3} (poles). The first pole b_{X2p1} is negative; the second pole b_{X2p2} is of particular interest, because it presents the left border of the permissible interval for b_{11} (the gray area in Table 2). From (21) the second and third poles of function X_2 follow: b_{X2p2}=2.709·10^{-6} and b_{X2p3}=3.491·10^{-6}.

Furthermore, the curve in Figure 2(c), represents the ratio J_2/J_3 and has one pole b_{X3p}, which follows from J_2=0, and zero b_{X3z}, from J_3=0. Since we need those characteristic points that are borders of the gray area in Table 2, only the zero b_{X3z} is of interest. The zero b_{X3z} is defined by:

$$b_{X3z} = \frac{b_{22}^2 b_{24} + b_{12}^2 b_{44} - b_{22}(b_{12} b_{34} + b_{44})}{b_{14} b_{22}^2 - b_{12} b_{44}} \ . \tag{22}$$

From (22) we obtain the value of b_{X3z}=2.9211·10^{-6}. Note that b_{X2p3}>b_{X3z}, therefore, the zero b_{X3z} presents the upper bound of the permissible interval for b_{11} in the case A. For the case B we obtain

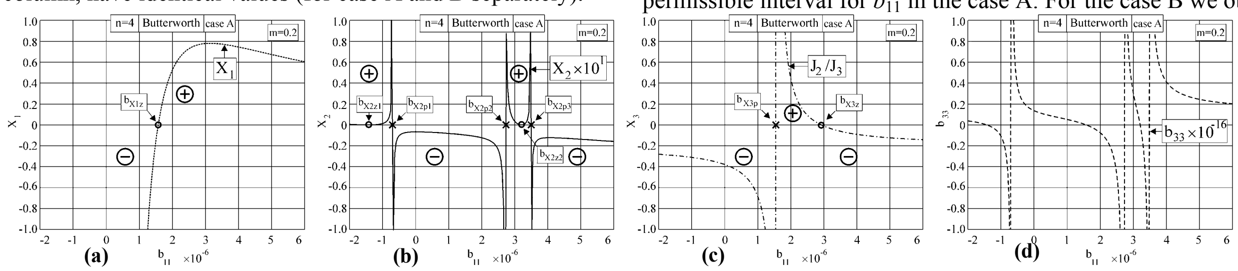

Figure 2. Functions X_1, X_2, J_2/J_3 and b_{33} vs. b_{11} (Butter., m=0.2, case A). Characteristic points and sign of: **(a)** X_1; **(b)** X_2; **(c)** X_3 and **(d)** b_{33}.

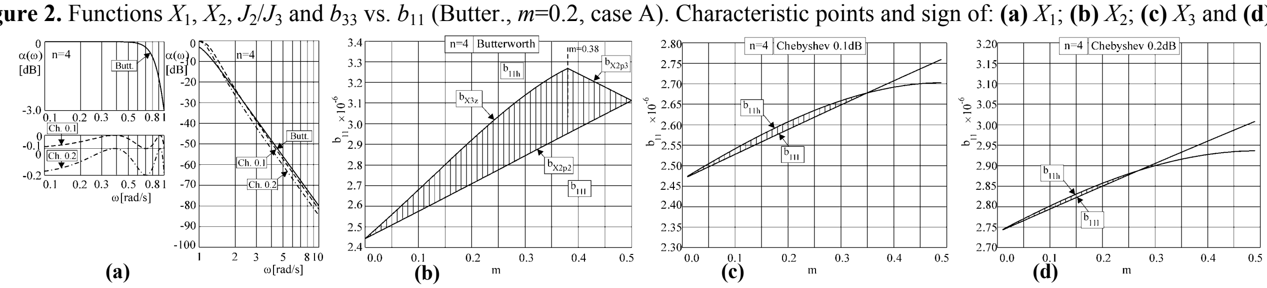

Figure 3. (a) Magnitudes of normalized TFs for filters in Table 1. **(b)-(d)** Permissible range of design parameter b_{11} vs. parameter m.

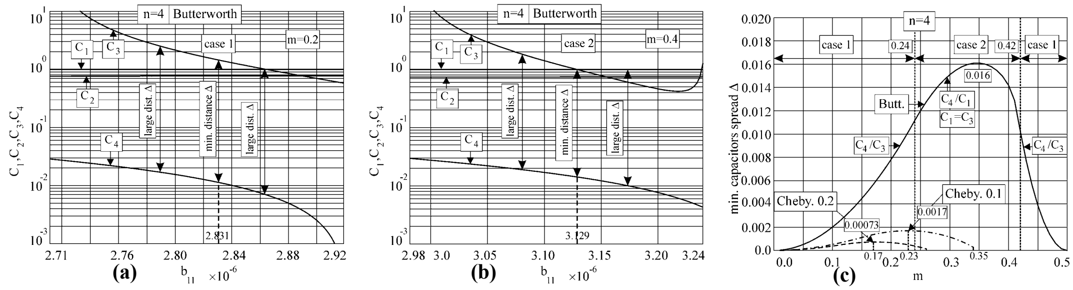

Figure 4. (a) Capacitor spread Δ, case 1, with m=0.2. **(b)** Capacitor spread Δ, case 2, with m=0.4. **(c)** Min. distance Δ vs. parameter m.

the following critical values: from (21) $b_{X2p2}=3.0056\cdot10^{-6}$, and $b_{X2p3}=3.2165\cdot10^{-6}$, and from (22) we have $b_{X3z}=3.3125\cdot10^{-6}$. Note that in this case: $b_{X3z}>b_{X2p3}$. Cases A and B differ in the upper bound b_{11h}. We can therefore derive the lower and upper bounds of the permissible interval for b_{11}; they are given by

$$b_{11l}=b_{X2p2}<b_{11}<\min\{b_{X2p3},b_{X3z}\}=b_{11h}\,. \qquad (23)$$

Observing lines 1 and 4 in Table 2 we can conclude that the zero of the function X_1 is always lower than the left bound in (23) and that the critical poles and zeros of the function b_{33} remain outside the bounds defined by (23), for any value of m. As a consequence of it, the equation (20) holds, the values X_1 and b_{33} are always positive, and do not need to be checked if the condition (23) is satisfied.

For the filter examples in Table 1, permissible ranges of b_{11} vs. parameter m, as defined by (23), are shown in Figure 3(b)-(d), from which we conclude that the Butterworth filter has the widest permissible range for values of b_{11}.

4.2. The Component Spread

For the practical realization of a filter, the problem of large differences in filter component values usually arises. In that sense the capacitor values are most problematical. The resistor values are very close, which causes no problems. Therefore, to choose an optimum design parameter m, we try to find the minimal spread of capacitor values.

We choose a normalized value of C_1=1, and in Figure 4(a), we present the values of capacitors C_1 to C_4 as a functions of parameter b_{11}, when m=0.2. According to (23) b_{11} takes values from $2.71\cdot10^{-6}$ to $2.92\cdot10^{-6}$ [s/rad]. From Figure 4(a) it is obvious that the best solution is for $b_{11}=2.71\cdot10^{-6}$, because the distance Δ between the largest C_3 and smallest capacitor C_4 in the filter circuit is then minimal. Note that the ordinate in Figure 4(a) is logarithmic, thus the distance $\Delta=\log C_3-\log C_4=\log(C_3/C_4)=\log(X_3^{-1})$. Therefore, we obtain the min. Δ for max. X_3. We distinguish between two cases related to the choice of the parameter m value, which are presented in Figure 4(a) and (b), respectively. In case 1 the ratio C_4/C_3 is the largest for all values b_{11}, while in case 2 the ratio C_4/C_1 exceeds the ratio of C_4/C_3. Consequently, in case 1 min. distance between largest and smallest capacitor is the max. of X_3, while in case 2 the minimum distance is a value of X_3 when $C_1=C_3$. When the condition, given by:

$$\max\{C_4/C_3=X_3\}<C_4/C_1=X_1X_2X_3 \text{ for } b_{11\max X3}\,, \quad (24)$$

is satisfied we have a case 1, otherwise we have a case 2. Note that both sides in condition (24) are calculated for the value of b_{11} for which X_3 reaches the maximum.

Finally, minimum capacitor distances Δ (or ratios) vs. parameter m are presented in the Figure 4(c). It is obvious that for the Butterworth filter no. 1) in Table 1 the best solution is for m=0.35, because the ratio $C_4/C_3=C_4/C_1=0.016$ is maximal, which means that the most critical capacitance C_4 is $(0.016)^{-1}=62.5$ times smaller than the largest capacitance in the circuit. For the Chebyshev filter no. 2) with 0.1[dB] pass-band ripple, the min. capacitor spread is 588 times (m=0.23), and for the filter no. 3) with 0.2[dB] the min. spread is even 1370 times (m=0.17). Thus, if we intended to realize the filter no. 3) on a chip, we would obtain the capacitance C_4 at the amplifier input so small that it would be comparable in value to the parasitic capacitance of the circuit. We conclude that the unity-gain 4th-order LP filter as in Figure 1, is mainly suitable for the realization of Butterworth filters.

5. CONCLUSION

It is well known that the 4th-order filter with gain β>1 cannot be solved analytically. In an attempt to design the β>1 filter, we would have to solve a set of non-linear equations by applying some *numerical* procedure (e.g. Newton's iterative method [5]), which can be done with modern filter design software. However, it may be of more than academic interest to find *analytical* solutions to the design of higher-than-third-order filters. In this paper it is shown that the active-*RC* 4th-order unity-gain low-pass filter can be designed analytically, provided the realizability constraints, which are presented, are satisfied. Furthermore, Newton's method can use the obtained β=1 filter elements as starting values [5] for filter design. The 4th-order filter with β=1 has limited practical significance because of severe constraints on realizability and very high capacitance ratios. Nevertheless, our new analysis and design method shows that the design of single-opamp 4th-order filters, having gain β≥1, is possible (see also [5]). The method provides useable results, and may be useful also for other than low-pass filters. In any case, when power is at premium, a fourth-order filter with only one amplifier may provide a very useful filter solution.

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